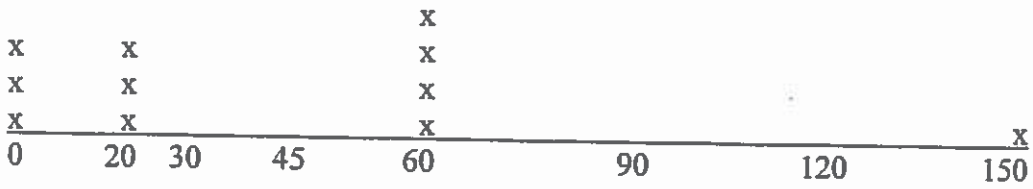
Name _____
Period _____

HOW LONG SHOULD THE DAILY LUNCH BREAK BE?

A small company has eleven employees. According to company policy, these employees will work four hours before lunch, four hours after, and all of them will be at lunch at the same time. However, the length of the lunch break can be whatever the employees and their manager decide.

The manager passes out a ballot with the following options for the length of the daily lunch break: 20 minutes, a half hour, 45 minutes, an hour, an hour-and-a half, two hours, and two-and-a-half hours. Each employee votes once by secret ballot. Here are the election results; the votes are marked by x's.

Election Results for Length of Daily Lunch Break



1) Suppose you were the manager. How long would you make the lunch break? _____ On what do you base your decision?

_____

_____

2) Now, let's say that, as the manager, you wanted to maximize the number of employees who get the exact lunch break length they voted for. How long would you make the lunch break? Why?

Is there a unique answer to this question for this data set? _____

Can you imagine vote results where the answer would not be unique? _____

3) Another way to look at the problem, is to consider the difference, for a given employee, between the length she voted for and the break length implemented, as her "unhappiness value." For example, if an employee wanted a 20 minute break but the company implemented a 60 minute break, her "unhappiness value," hereafter abbreviated "uv," is abs (20 – 60) = 40 minutes. If someone wanted a 90 minute break

Key

but a 30 minute break was implemented, his uv is abs (90 – 30) = 60 minutes. Then the question becomes, what length will minimize the total uv for these eleven employees? Try different lengths by guess and check. Hint: The answer is one of the ballot options.

. If we implement this length, what is the sum of the uv?

4) Let's say we want to be certain that we answered Prob. 3 correctly. A thorough method is to create a function whose x-values vary over the domain of lunch break lengths. The function values y will be the sum of the absolute value differences. We will graph the function and then find its minimum.

Set up your WINDOW: Xmin = 0, Xmax = 94, Xscl = 10, Ymin = 300, Ymax = 600, Yscl = 50, Xres = 1.

Note: To produce the absolute value command abs: Hit MATH. Arrow right to NUM. Hit 1.

Input the Y-function: Hit Y=. Input $Y_1 = 3 * abs(X - 0) + 3 * abs(X - 20) + 4 * abs(X - 60) + abs(X - 150)$.

Hit GRAPH. Hit TRACE. Arrow right or left until you reach the minimum. The minimum uv sum of _____, the y-value, occurs at x = _____ Was this your answer to Prob. 3? Is there a unique answer to this question for this data set? _____ Can you imagine vote results that would not have a unique answer in minimizing the sum of the uv's? _____

5) In Prob. 2, we maximized the number of employees with the exact lunch break they voted for. In Probs. 3 and 4, we minimized the total unhappiness value uv. Let's look at yet a third solution.

Consider that if a given employee's uv is quite high, they may quit. To avoid this, we may want to give great effect to the larger uv's by summing the squares of these values. Look at the vote chart from the previous page, especially the vote for a 150 minute lunch. Compared to your answer to Prob. 3 above, do you think the lunch break length that minimizes the sum *of the squares* of the uv's is longer or shorter than your Prob. 3 answer? _____ Why? _____

6) To solve Prob. 5 graphically, set your WINDOW to: Xmin = 0, Xmax = 94, Xscl = 10, Ymin = 15000, Ymax = 50000, Yscl = 5000, Xres = 1. Then input the function that minimizes the sum of the squares of the differences:

Hit Y= Input $Y_2 = 3 * (X - 0)^2 + 3 * (X - 20)^2 + 4 * (X - 60)^2 + (X - 150)^2$.

Hit GRAPH. Hit TRACE. Arrow down once. (onto the Y_2 function.) Arrow right or left until you reach the minimum. To find the true minimum:

Hit 2nd CALC. Hit "3" (the minimum). Arrow down once. (onto the Y_2 function.) The calculator queries "Left Bound?" Input 30. Hit ENTER. It says, "Right Bound?"

Key

Input 50. Hit ENTER twice. The minimum of the sum of squares of the uv's of the y-value, occurs at x = _____. Is the answer to this question unique for this data set? _____ Will there always be a unique answer for any set of vote results? _____ Why or why not? _____

Now, let's consider a new set of eight employees. Their votes for lunch break length were: 0, 0, 30, 45, 90, 90, 120, and 120 minutes. Let's rework Probs. 2, 4 and 6, respectively as Probs. 7, 8, and 9 using the new data.

7) What lunch break length maximizes the number of employees who get the exact length they voted for? _____ Is the answer unique for this data set? _____ Why or why not? _____

8) What lunch break length minimizes the sum of the uv's? _____ Hint: Post a function similar to that of Prob. 4 into Y_1. This time use the WINDOW: Xmin = 0, Xmax = 141, Xscl = 20, Ymin = 300, Ymax = 500, Yscl = 100. The sum of the uv's = _____ Is it unique for this data set? _____ Why or why not? _____

9) What lunch break length minimizes the sum of squares of the uv's? _____ Hint: Post a function similar to that of Prob. 6 into Y_2. This time use WINDOW: Xmin = 0, Xmax = 141, Xscl = 20. Ymin = 0, Ymax = 50000, Yscl = 5000. The sum of the squares of the uv's = _____ Is it unique for this data set? _____ Why or why not? _____

Shortcut Solution and Summary

10) Find the mode, the median, and mean for the original eleven employees. Find these three centers of data for the later eight employees:

	Optimum Solution for:	Original Eleven	All Solutions	Later Eight	All Solutions	Always Unique?
Mode (s)						
Median						
Mean						